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CONVECTION OF A BINARY MIXTURE UNDER CONDITIONS OF THERMAL DIFFUSION AND VARIABLE TEMPERATURE GRADIENT

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Instability of a plane horizontal layer of an incompressible binary gas mixture stratified in the gravity field under the action of a transverse temperature gradient modulated in time is studied. The case of solid impermeable boundaries of the layer, where the flux of matter vanishes, is considered. The analysis is based on the Floquet method applied to linearized equations of convection in the Boussinesq approximation. It is shown that there are regions of parametric instability at finite frequencies. In addition to the synchronous or subharmonic response to an external action, the instability may be related to quasi-periodic disturbances. Depending on the amplitude and frequency, modulation can stabilize the unstable basic state and also destabilize the equilibrium of the fluid. The threshold values of convection for modulations of temperature and translational vertical vibrations are compared.

Introduction. The presence of a variable parameter in a hydrodynamic system can significantly affect its stability [1], which is used to control fluid motion in various technological processes. Variable electric and magnetic fields, temperature gradients, or vibrations are various means of periodic actions on mechanical systems, in particular, fluids.

The effect of modulation of the boundary temperature on convective instability of a horizontal layer of a fluid was studied in [1–3]. The basic state of the system is quasi-equilibrium, where the fluid is stationary, and a heat wave propagates between the boundaries of the layer. The instability of this state may be related to two types of critical disturbances. The period of disturbances of the first type coincides with the period of the external action (synchronous response of the system), and the period of disturbances of the second type is twice as large (subharmonic response).

If there are vibrational modes of instability in a convective system in the absence of modulation of external fields, a variable action leads to origination of a new type of disturbances — quasi-periodic disturbances characterized by two frequencies: the frequency of the external field and the modified eigenfrequency of neutral oscillations. An example of convective systems with a vibrational mode of instability is a binary mixture in the range of parameters, where the anomalous effect of the Soret thermal diffusion is manifested. The influence of transverse translational vibrations on convective stability of a binary mixture was studied earlier [4] for high frequencies, where the amplitude and frequency of oscillations of the system are not independent parameters. In the case of finite frequencies of vibrations, the instability can be caused by resonance effects [5].

In the present paper, we consider a parametric action on a horizontal layer of a binary mixture with thermal diffusion. The case of emergence of convection in a constant gravity field in the presence of a variable temperature difference at the boundaries is examined.

1. Formulation of the Problem. We consider a binary mixture that fills a plane horizontal layer bounded by perfectly heat-conducting solid impermeable parallel planes $z = \pm h$ (h is the half-thickness of the layer), where different temperatures that vary as $T(\pm h) = \mp\Theta(\eta_1 + \eta_2 \cos \Omega t)$ are sustained. Here Θ is the characteristic scale of temperature, η_1 and η_2 are the relative amplitudes of the constant and variable components of the temperature difference at the boundaries, Ω is the cyclic modulation frequency, and t is the time. In the case considered, η_1 can take two values: $\eta_1 = 0$ for temperature modulation at the boundaries with a zero mean value and $\eta_1 = 1$ for

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modulation at a constant background. The cases $\Theta > 0$ and $\Theta < 0$ correspond to heating from below and from above, respectively. In the present problem, the concentration gradient even in the initially homogeneous mixture is formed because of the temperature gradient and Soret thermal diffusion.

We write the equation of state of the binary mixture in the form

$$\rho = \bar{\rho}(1 - \beta_T T - \beta_C C),$$

where $\bar{\rho}$ is the density of the mixture at mean values of temperature and concentration, T and C are small deviations of temperature and concentration from the mean values, and β_T and β_C are the coefficient of thermal expansion and the concentration coefficient of density (if C is the concentration of the light component, then $\beta_C > 0$).

To nondimensionalize the variables, we introduce the scales of the distance h, time h^2/ν, velocity ν/h, temperature Θ, concentration $\beta_T\Theta/\beta_C$, and pressure $\bar{\rho}\nu^2/h^2$ (ν and χ are the kinematic viscosity and temperature diffusivity).

The dimensionless system of equations of convection for the binary mixture in the Boussinesq approximation acquires the form

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\nabla)\boldsymbol{v} = -\nabla p + \Delta \boldsymbol{v} + \mathrm{Gr}(T + C)\boldsymbol{n},$$

$$\frac{\partial T}{\partial t} + \boldsymbol{v}\nabla T = \frac{1}{\mathrm{Pr}}\,\Delta T, \qquad \operatorname{div} \boldsymbol{v} = 0, \qquad \boldsymbol{n} = (0, 0, 1),$$

$$\frac{\partial C}{\partial t} + \boldsymbol{v}\nabla C = \frac{1}{\mathrm{Sc}}\,\Delta(C - \varepsilon T),$$

where $\boldsymbol{v}$ is the velocity, p is the pressure, $\mathrm{Gr} = g\beta_T\Theta h^3/\nu^2$, $\mathrm{Pr} = \nu/\chi$, and $\mathrm{Sc} = \nu/D$ are the Grashof, Prandtl, and Schmidt numbers, respectively, $\varepsilon = -\beta_C\alpha/\beta_T$ is the Soret parameter, D and α are the coefficients of diffusion and thermal diffusion, and $\omega = \Omega h^2/\nu$ is the dimensionless modulation frequency.

In studying the convection of the binary system, we considered different variants of boundary conditions. It seems that the test conditions are in best agreement with the case of impermeable solid boundaries [6], where the flux of matter vanishes:

$$z = \pm 1: \qquad \boldsymbol{v} = 0, \quad T = \mp(\eta_1 + \eta_2 \cos \omega t), \quad \frac{\partial C}{\partial z} - \varepsilon\,\frac{\partial T}{\partial z} = 0. \tag{1}$$

The problem admits a quasi-equilibrium solution, where the fluid is at rest ($\boldsymbol{v}_0 = 0$), and its remaining characteristics vary in time and space: $T_0 = T_0(z, t)$, $p_0 = p_0(z, t)$, and $C_0 = C_0(z, t)$. In what follows, the explicit expression for the pressure p_0 is not used. The unsteady distributions of temperature $T_0(z, t)$ and concentration $C_0(z, t)$ satisfy the one-dimensional equations of heat conduction and diffusion

$$\mathrm{Pr}\,\frac{\partial T_0}{\partial t} = \frac{\partial^2 T_0}{\partial z^2}, \qquad \mathrm{Sc}\,\frac{\partial C_0}{\partial t} = \frac{\partial^2 C_0}{\partial z^2} - \varepsilon\,\frac{\partial^2 T_0}{\partial z^2}$$

and the corresponding boundary conditions (1). The temperature distribution under quasi-equilibrium conditions is determined by superposition of the linear profile and two heat waves propagating from the boundaries inside the fluid. The concentration distribution is determined by the temperature distribution due to the thermal diffusion effect:

$$T_0 = -\eta_1 z - \mathrm{Re}\Big[(\eta_2 \sinh qz/\sinh q)\exp(i\omega t)\Big],$$

$$C_0 = -\eta_1\varepsilon z + \mathrm{Re}\Big[\frac{\varepsilon\eta_2}{(q^2 - r^2)\sinh q}\Big(\frac{qr\cosh q \sinh rz}{\cosh r} - q^2\sinh qz\Big)\exp(i\omega t)\Big], \tag{2}$$

$$q = (1 + i)\sqrt{\omega\mathrm{Pr}/2}, \qquad r = (1 + i)\sqrt{\omega\mathrm{Sc}/2}.$$

To study the stability of the basic state (2), we consider its small perturbations $\boldsymbol{v}$, T', C', and p' and introduce a new variable $H' = C' - \varepsilon T'$. After linearization, we obtain a system of equations and boundary conditions for disturbance evolution:

$$\frac{\partial \boldsymbol{v}}{\partial t} = -\nabla p + \Delta \boldsymbol{v} + \mathrm{Gr}(T'(1 + \varepsilon) + H')\boldsymbol{n}, \qquad \frac{\partial T'}{\partial t} + \boldsymbol{v}\nabla T_0 = \frac{1}{\mathrm{Pr}}\,\Delta T',$$

$$\frac{\partial H'}{\partial t} + \boldsymbol{v}(\nabla C_0 - \varepsilon\nabla T_0) = \frac{1}{\mathrm{Sc}}\,\Delta H' - \frac{\varepsilon}{\mathrm{Pr}}\,\Delta T', \qquad \operatorname{div} \boldsymbol{v} = 0,$$

$$z = \pm 1: \qquad \boldsymbol{v} = 0, \quad T' = 0, \quad \frac{\partial H'}{\partial z} = 0.$$

Eliminating the pressure and horizontal components of velocity, the perturbations of the vertical velocity v_z, temperature T', and function H' are written as

$$\begin{pmatrix} v_z \\ T' \\ H' \end{pmatrix} = \begin{pmatrix} w(z,t) \\ \theta(z,t) \\ \xi(z,t) \end{pmatrix} \exp{(ik_x x + ik_y y)},$$

where w, θ, and ξ are the amplitudes and $\boldsymbol{k}$ is the wave vector that characterizes the periodicity of disturbances in the plane of the layer ($k^2 = {k_x}^2 + {k_y}^2$).

For disturbance amplitudes, we obtain the problem

$$\begin{gathered} \frac{\partial Dw}{\partial t} = D^2 w - k^2 \mathrm{Gr}(\theta(1+\varepsilon) + \xi), \qquad \frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}}\, D\theta - w\nabla T_0, \\ \frac{\partial \xi}{\partial t} = \frac{1}{\mathrm{Sc}}\, D\xi - \frac{\varepsilon}{\mathrm{Pr}}\, D\theta - w(\nabla C_0 - \varepsilon \nabla T_0), \qquad D = \frac{\partial^2}{\partial z^2} - k^2. \end{gathered} \tag{3}$$

The boundary conditions for the amplitudes on the solid isothermal planes are

$$z = \pm 1: \qquad w = 0, \quad w' = 0, \quad \theta = 0, \quad \xi' = 0, \tag{4}$$

where the prime denotes the derivative with respect to the transverse coordinate z.

System (3), boundary conditions (4), and conditions of periodicity in time for all variables determine the eigenvalue problem for the Grashof number as a function of the remaining parameters. The boundaries of convective instability determined by the conditions of existence of periodic solutions of system (3) can be found using the classical Floquet method.

2. Method of the Solution. The temperature and concentration gradients in the basic state are even functions of the vertical coordinate z. Hence, the eigenfunctions of problem (3) are divided into two classes: odd and even in terms of z. It is known that "one-storeyed" disturbances corresponding to even eigenfunctions are most unstable [1]; therefore, the disturbances are approximated by even spatial basis functions with time-dependent coefficients:

$$w = \sum_{m=0}^{M-1} a_{2m}(t) w_{2m}, \qquad \theta = \sum_{m=0}^{M-1} b_{2m}(t)\theta_{2m}, \qquad \xi = \sum_{m=0}^{M-1} c_{2m}(t)\xi_{2m}. \tag{5}$$

As the basis functions for the vertical velocity, temperature, and concentration, we use the eigenfunctions of the fourth- and second-order boundary problems

$$D^2 w_{2m} = -\mu_{2m} D w_{2m}, \qquad w_{2m}(\pm 1) = w'_{2m}(\pm 1) = 0,$$

$$\mathrm{Pr}^{-1} D\theta_{2m} = -\nu_{2m}\theta_{2m}, \qquad \theta_{2m}(\pm 1) = 0,$$

$$\mathrm{Sc}^{-1} D\xi_{2m} = -\rho_{2m}\xi_{2m}, \qquad \xi'_{2m}(\pm 1) = 0,$$

where μ_{2m}, ν_{2m}, and ρ_{2m} are the eigenvalues for the corresponding basis functions. For the temperature θ_{2m} and concentration ξ_{2m}, we have trigonometric basis functions. The functions w_{2m} proposed in [7] form a full orthnormalized system

$$\int_{-1}^{1} w_i D w_j \, dz = -\delta_{ij}.$$

Substituting expansions (5) into system (3) and performing orthogonalization by the Galerkin method, we obtain $K = 3M$ ordinary differential equations for a_r, b_s, and c_t of the form

$$\frac{\partial u_i}{\partial t} = L_{ij}(\omega t) u_j, \quad i,j = 3M, \qquad \boldsymbol{u}(t) = \begin{pmatrix} a_r \\ b_s \\ c_t \end{pmatrix}, \tag{6}$$

where the matrix L is periodic with a period $2\pi/\omega$ and $\boldsymbol{u}(t)$ is a K-dimensional vector function. According to the classical Floquet theory [8], all solutions of system (6) can be written as

$$\boldsymbol{u}(t) = \mathrm{e}^{\lambda t}\boldsymbol{u}_0(t) = \gamma \boldsymbol{u}_0(t),$$

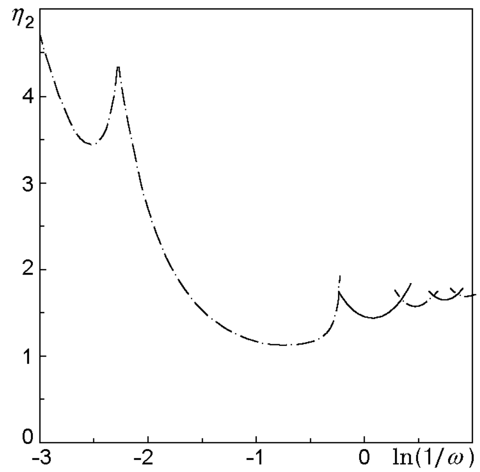

Fig. 1. Amplitude of modulation η_2 as a function of the inverse frequency in the absence of the mean temperature gradient ($\eta_1 = 0$): the dot-and-dashed and solid curves show the boundaries of stability for subharmonic disturbances and synchronous regimes, respectively.

where $\boldsymbol{u}_0(t)$ is a vector with a period $2\pi/\omega$. Here, γ is called the Floquet multiplier and $\lambda = \lambda_r + i\lambda_i$ is called the characteristic index. For different independent initial conditions $u_i^p(0) = \delta_{ip}$ ($p = 1, \ldots, K$), system (6) is integrated by the fourth-order Runge–Kutta method. The fundamental solutions taken at the end of the modulation period $u_i^p(t)$ compose K columns of the monodromy matrix with the dimension $K \times K$ whose eigenvalues are the Floquet multipliers. The values of characteristic indices determine the stability of the basic quasi-equilibrium state. If λ_i is ordered so that $\mathrm{Re}\,(\lambda_1) \geqslant \mathrm{Re}\,(\lambda_2) \geqslant \ldots \geqslant \mathrm{Re}\,(\lambda_K)$, then the basic state is stable for $\mathrm{Re}\,(\lambda_1) < 0$. The condition $\mathrm{Re}\,(\lambda_1) = 0$ determines the range of existence of periodic solutions in the space of parameters Gr, η_1, η_2, Pr, Sc, ε, ω, and k. The case $\mathrm{Re}\,(\lambda_1) = 0$, $\mathrm{Im}\,(\lambda_1) = \omega/2$ corresponds to subharmonic disturbances with a period twice as large as the period of the external action. If $\mathrm{Re}\,(\lambda_1) = 0$, $\mathrm{Im}\,(\lambda_1) = \omega$, then the neutral disturbances vary simultaneously with the forcing action, and their periods coincide. The pair of complex-conjugate eigenvalues with a unit modulus [$\mathrm{Re}\,(\lambda_1) = 0$, $\mathrm{Im}\,(\lambda_1) \neq 0$] correspond to quasi-periodic neutral disturbances. For most solutions found, 15 basis functions were used ($M = 5$). In test computations performed with 21 basis functions ($M = 7$), convection thresholds varied by less than 1%. The following values of the Prandtl and Schmidt numbers characterizing the gas mixture were used in all computations: Pr $= 0.75$ and Sc $= 1.5$.

3. Results of the Analysis. In the case of a constant temperature gradient, there are different regions of instability on the plane $(\varepsilon, \mathrm{Gr})$. In the absence of thermal diffusion ($\varepsilon = 0$ for Pr $= 0.75$), the equilibrium is violated at $\mathrm{Gr}_{\mathrm{cr}} = 142.37$, which corresponds to the Rayleigh number for the layer with solid boundaries ($\mathrm{R} = 16\mathrm{Gr}_{\mathrm{cr}}\mathrm{Pr} = 1708.5$) determined by the layer thickness and temperature difference at the boundaries. In the case of heating from below, the region of monotonic instability corresponds to the value $\varepsilon > -0.1$ and the region of vibrational instability to $\varepsilon < -0.1$. Cellular disturbances are the most dangerous in this case [6]. For $\varepsilon = -0.3$, the convection threshold ($\mathrm{Gr}_{\mathrm{cr}} = 271.686$) corresponds to the critical wavenumber $k_{\mathrm{cr}} = 1.357$ and the frequency $\omega_0 = 2.665$. For heating from above (Gr < 0), the instability is possible in the case of anomalous thermal diffusion ($\varepsilon < 0$); long-wave monotonic disturbances are critical.

The thresholds of instability of thermoconcentration convection in a variable thermal field as a result of minimization in terms of the wavenumber k are shown in Figs. 1–5.

First, we consider the case of modulation with respect to the zero mean value of temperature ($\eta_1 = 0$). The modulation amplitude η_2 corresponding to the stability boundary is plotted in Fig. 1 as a function of the inverse frequency $\ln(1/\omega)$ for a fixed value Gr $= 260$. Without modulation, the system is stable: $\mathrm{Gr} < \mathrm{Gr}_{\mathrm{cr}} = 271.686$. Hereinafter, the dot-and-dashed and solid curves refer to the boundaries of stability for subharmonic regimes and growing synchronous disturbances, respectively. The boundaries of instability of different types intersect in the chart of stability; the first two regions of instability are associated with subharmonic disturbances, and a further decrease in frequency leads to alternation of the regions of synchronous and subharmonic instability. The mechanism of destabilization includes the resonant interaction of the least stable vibrational mode with an eigenfrequency ω_0 and

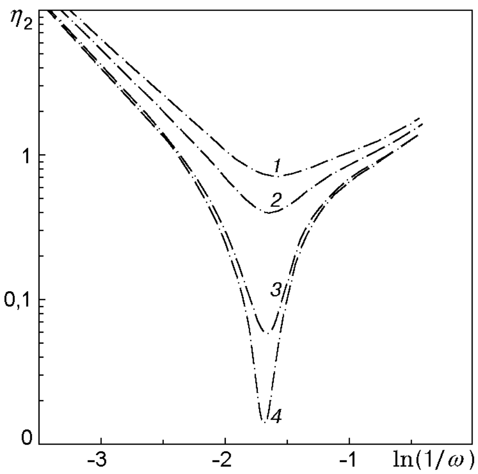

Fig. 2

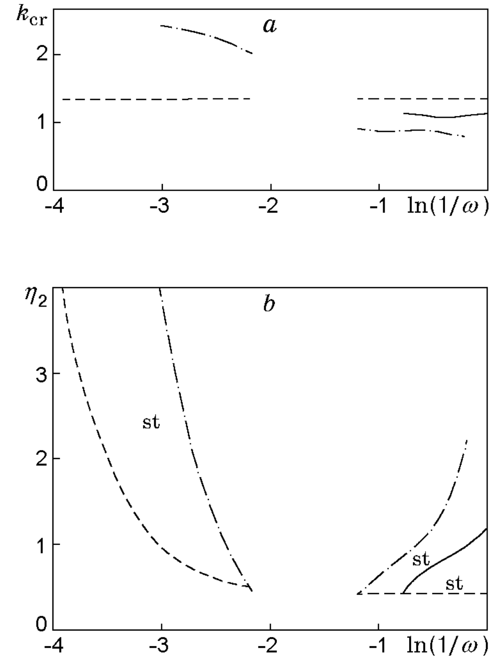

Fig. 3

Fig. 2. Modulation amplitude η_2 versus the inverse frequency in the first resonant region of the subharmonic response in the presence of the mean temperature gradient ($\eta_1 = 1$) for Gr = 180 (1), 210 (2), 260 (3), and 269 (4).

Fig. 3. Critical wavenumbers $k_{\rm cr}$ (a) and modulation amplitude η_2 (b) versus the inverse frequency for $\eta_1 = 1$: the dashed curves are the boundaries of quasi-periodic regimes, the solid curves are the boundaries of stability for synchronous regimes, and the dot-and-dashed curves are the boundaries of stability for subharmonic disturbances.

temperature oscillations in a modulated thermal field. The greatest destabilization corresponds to the amplitude $\eta_1 = 1.117$ and is observed for the frequency ratio $\omega = 2.0 \simeq \omega_0$. In the absence of the constant component of the temperature gradient, the governing parameter is the product η_2Gr. Thus, it is possible to obtain stability charts for all values of Gr.

A nonzero constant component of the temperature gradient ($\eta_1 = 1$) is responsible for the independence of the regime parameters Gr and η_2. The modulation amplitude η_2 is plotted in Fig. 2 as a function of the inverse frequency $\ln(1/\omega)$ for various values of "subcritical" heating ($\mathrm{Gr} < \mathrm{Gr}_{\rm cr}$). In the absence of modulation ($\eta_2 = 0$), the quasi-equilibrium of the fluid is stable. An increase in the modulation amplitude leads to emergence of growing disturbances. The type of critical disturbances depends on frequency. The first resonant region, where the effect of destabilization is manifested most clearly, corresponds to subharmonic disturbances relative to the external action; its minimum is located at the frequency $\omega \simeq 5.33 \simeq 2\omega_0$. To excite convection parametrically at high values of the parameter $(\mathrm{Gr}_{\rm cr} - \mathrm{Gr})/\mathrm{Gr}_{\rm cr}$, one has to increase the modulation amplitude.

The behavior of the critical wavenumbers $k_{\rm cr}$ and modulation amplitudes at the stability boundary in the case of "supercritical" heating ($\mathrm{Gr} = 280 > \mathrm{Gr}_{\rm cr}$) is shown in Fig. 3. Without modulation, the quasi-equilibrium is unstable. An increase in the modulation amplitude leads to the appearance of stability regions (st) on the plane $(\mathrm{Gr}, 1/\omega)$, which are symmetric about the resonant frequency $\omega = 5.33 \simeq 2\omega_0$. The lower boundary of these domains is determined by the fundamental mode of quasi-periodic instability and corresponds to the maxima in the curves $k_{\rm cr} = k_{\rm max}$. The upper boundary of the stability regions is caused by parametric effects and corresponds to the minima in the curves $k_{\rm cr} = k_{\rm min}$ with subharmonic or synchronous disturbances growing above it. A competition of these modes is observed at comparatively low frequencies. In the region of stability relative to subharmonic disturbances, a domain of synchronous instability appears. It is seen in Fig. 3 that modulation stabilizes the basic state in a wide range of frequencies, but parametric instability is excited in the cases $\omega \simeq 2\omega_0$ and $\omega \simeq \omega_0$. With increasing modulation amplitude, the range of resonant excitation increases.

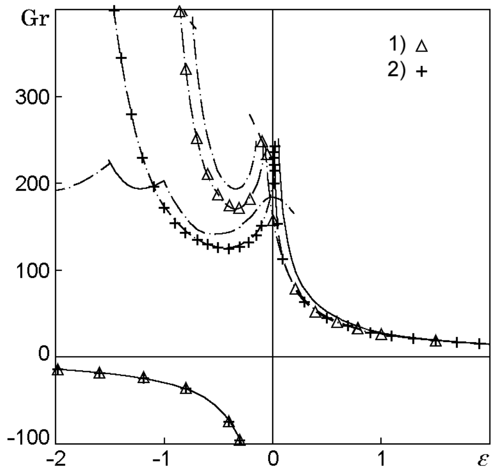

Fig. 4. Charts of stability in the plane $(\varepsilon, \mathrm{Gr})$ in the presence of vibrations [5] and in a modulated thermal field ($\omega = 2\pi$ and $\eta_1 = 1$) for $\eta_2 = 1$ (1) and 2 (2); the remaining notation the same as in Fig. 3.

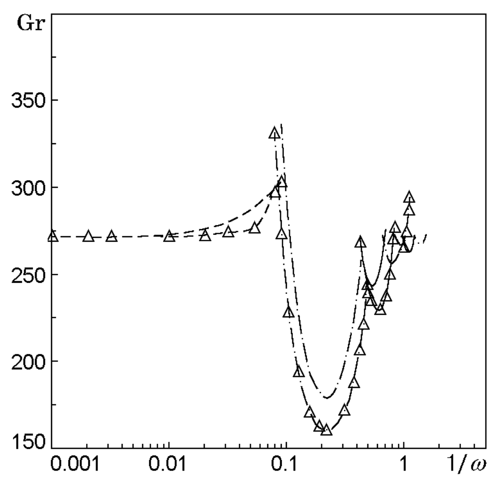

Fig. 5

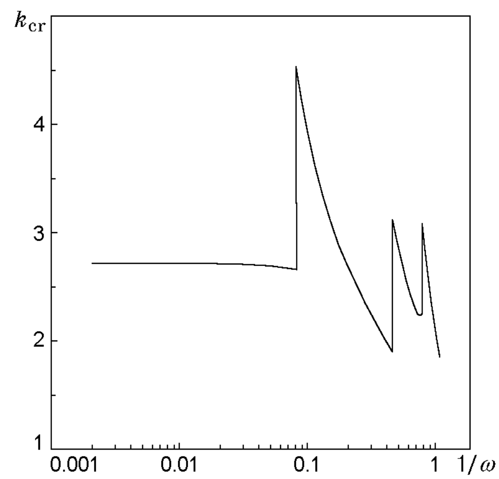

Fig. 6

Fig. 5. Charts of stability in the plane $(1/\omega, \mathrm{Gr})$ in a modulated field ($\varepsilon = -0.3$, $\eta_1 = 1$, and $\eta_2 = 1$): the points refer to temperature modulation; the remaining notation the same as in Fig. 3.

Fig. 6. Critical wavenumbers $k_{\rm cr}$ versus $1/\omega$ in a modulated thermal field ($\varepsilon = -0.3$, $\eta_1 = 1$, and $\eta_2 = 1$).

Figure 4 shows the boundaries of convective instability $\mathrm{Gr}(\varepsilon)$ for various methods of parametric actions on the binary mixture (vertical vibrations of the layer [5] and modulation of the temperature gradient). The modulation frequency is fixed: $\omega = 2\pi$, and the modulation amplitude takes the values $\eta_2 = 1$ and 2. The dimensionless amplitude of vibrations is $\eta_2 = b\Omega^2/g$ (b is the amplitude and Ω is the cyclic frequency of vibrations). The thresholds of long-wave disturbances existing in the case of heating from above remain almost constant with changing the amplitude η_2 and the method of the parametric action. This is related to specific features of the action of the thermoconcentration (double diffusion) mechanism of instability in the binary mixture, which is caused by the difference in characteristic times of heat conduction and diffusion.

In the region of the normal effect of thermal diffusion ($\varepsilon > 0$), temperature modulation has almost no effect on the boundary of stability either; high-amplitude vibrations stabilize the quasi-equilibrium. In the case of an anomalous Soret effect, temperature modulation exerts a stronger destabilizing effect on the quasi-equilibrium in the region $\varepsilon > -1$; the instability is related to subharmonic disturbances. In the region $\varepsilon < -1$, vibrations with a rather high amplitude ($\eta_2 = 2$) have a greater destabilizing effect on the binary mixture than modulation of the temperature gradient.

The thresholds of convection in the plane $(1/\omega, \mathrm{Gr})$ for two types of the parametric action are shown in Fig. 5 ($\varepsilon = -0.3$, $\eta_1 = 1$, and $\eta_2 = 1$). Figure 6 shows the behavior of the critical wavenumbers k_{cr} for the case of temperature modulation. The wavenumbers decrease monotonically inside each region of instability. At the points of intersection of instability boundaries, a competition of two modes with different spatial periods is observed. At high frequencies ($\omega > 1.25$), temperature modulation has a stronger destabilizing effect on the quasi-equilibrium; at low frequencies ($\omega < 1.25$), the parametric instability under the action of vibrations occurs at lower values of Gr.

Conclusions. The problem of convective instability of a nonuniformly heated binary mixture with allowance for the effect of thermal diffusion under the action of modulation of the transverse temperature gradient of an arbitrary frequency is considered on the basis of the Boussinesq equations. At finite modulation frequencies, both destabilization and stabilization of equilibrium are possible, depending on the characteristics of the parametric action. The instability of the fluid may be caused by disturbances with different time dependences, which correspond to a synchronous or subharmonic response to the external action or to quasi-periodic regimes. The boundaries of instability regions are determined.

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